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DIELECTRIC AND PERMEABILITY EFFECTS IN COLLISIONLESS PLASMA

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by

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Abstract

Using the unabridged Maxwell equations (including \underline{D} , \underline{E} , \underline{B} and \underline{H}) new effects in collisionless plasmas are uncovered. In a steady state, it is found that spatially varying energy density of the electric field (\underline{E}_\perp) orthogonal to \underline{B} produces electric current leading, under certain conditions, to the relationship

$$p_\perp + \frac{B^2}{8\pi} - \frac{\epsilon E_\perp^2}{8\pi} = \text{constant}$$

where ϵ is the dielectric constant of the plasma for fields orthogonal to \underline{B} . In steady state quasi-two-dimensional flows in plasmas, a general relationship between the components of electric field parallel and perpendicular to \underline{B} is found. These effects are significant in geophysical and astrophysical plasmas. The general conditions for a steady state in a collisionless plasma are deduced. With time variations in a plasma, slow compared to ion-gyro-period, there is a general current, (\underline{j}^*), which includes the well-known polarisation current, given by

$$\underline{j}^* = \left[\frac{\partial}{\partial t} (\underline{E} \times \underline{M}) + (\underline{P} \times \underline{B}) \right] \times \underline{B} B^{-2}$$

where \underline{M} and \underline{P} are the magnetisation and polarisation vectors respectively.

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Introduction

In this paper, macroscopic processes in a collisionless plasma are considered from the standpoint of Maxwell's unabridged equations including all four vectors \underline{D} , \underline{E} , \underline{B} and \underline{H} . This is in contrast to the widespread approach which employs \underline{E} and \underline{B} , but not \underline{D} and \underline{H} . New results are obtained which are not evident in the conventional approach. The paper starts with Maxwell's unabridged equations and states, as is well-known, how these are often abbreviated for use in methods which frequently employ velocity distribution functions of particles. After uncovering new effects in collisionless plasma using general arguments from Maxwell's equations, the approach of this paper is placed in context in plasma physics, and is shown how it extends our understanding of plasmas.

Maxwell's equations as commonly written are, in c.g.s. units,

$$\underline{\nabla} \times \underline{E} = - \frac{1}{c} \frac{\partial \underline{B}}{\partial t}, \quad (1)$$

$$\underline{\nabla} \times \underline{H} = \frac{1}{c} \frac{\partial \underline{D}}{\partial t} + \frac{4\pi \underline{J}}{c}, \quad (2)$$

where \underline{E} = electric field, i.e., electric force per unit charge,

\underline{H} = magnetic field,

$$\underline{B} = \text{magnetic induction} = \underline{H} + 4\pi \underline{M}, \quad (3)$$

\underline{M} = magnetic dipole moment per unit volume (magnetisation vector),

$$\underline{D} = \text{the electric displacement} = \underline{E} + 4\pi \underline{P}, \quad (4)$$

\underline{P} = electric dipole moment per unit volume (polarisation vector),

\underline{J} = total current excluding displacement current (see Maxwell, 1891).

Following Maxwell,

$$\underline{\nabla} \cdot \underline{D} = 4\pi \rho. \quad (5)$$

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From equation 1 $\frac{\partial}{\partial t} (\underline{\nabla} \cdot \underline{B}) = 0$, from which it is usually assumed that (Stratton, 1941)

$$\underline{\nabla} \cdot \underline{B} = 0. \quad (6)$$

From equation 2 it follows that $4\pi \operatorname{div} \underline{J} + \frac{\partial}{\partial t} \operatorname{div} \underline{D} = 0$, and

$$\underline{\nabla} \cdot \underline{J} + \frac{\partial \rho}{\partial t} = 0. \quad (7)$$

Either one accepts 5 to infer 7 or uses the conservation of charge (equation 7) to infer 5 (Stratton, 1941).

Maywell's equations may be presented in other forms (Stratton, 1941) e.g.,

$$\underline{\nabla} \times \underline{E} = - \frac{1}{c} \frac{\partial \underline{B}}{\partial t}, \quad (8)$$

$$\underline{\nabla} \times \underline{B} = \frac{1}{c} \frac{\partial \underline{E}}{\partial t} + \left(\frac{4\pi}{c} \frac{\partial \underline{P}}{\partial t} + 4\pi \underline{\nabla} \times \underline{M} + \frac{4\pi}{c} \underline{J} \right), \quad (9)$$

$$\underline{\nabla} \cdot \underline{B} = 0 \quad (10)$$

$$\underline{\nabla} \cdot \underline{E} = 4\pi\rho - 4\pi \underline{\nabla} \cdot \underline{P} \quad (11)$$

In many plasma physics books and papers (e.g. Spitzer, 1962) instead of equation 9, there is written

$$\underline{\nabla} \times \underline{B} = \frac{1}{c} \frac{\partial \underline{E}}{\partial t} + \frac{4\pi}{c} \underline{j}, \quad (12)$$

where, therefore,

$$\underline{j} = \frac{\partial \underline{P}}{\partial t} + c \underline{\nabla} \times \underline{M} + \underline{J} \quad (13)$$

and instead of equation 11 is written

$$\underline{\nabla} \cdot \underline{E} = 4\pi\rho_E, \quad (14a)$$

where, therefore,

$$\rho_E = \rho - \underline{\nabla} \cdot \underline{P} \quad (14b)$$

for a plasma consisting of electrons (-) and one kind of ion (+)

$$\underline{j} = e (N^+ V^+ - N^- V^-), \quad (15a)$$

$$\rho_E = (N^+ - N^-) e,$$

$$\underline{V} = \frac{1}{N} \int \underline{v} f(\underline{r}, \underline{v}, t) d^3v, \quad (15b)$$

and

$$\underline{N} = \int f(\underline{r}, \underline{v}, t) d^3v. \quad (16)$$

It is argued in texts on plasma physics that the velocity distribution f takes into account all the motions of all the particles and hence equation 14a and integral 14b gives the quantity \underline{j} , then using integral 15 to find N^+ and N^- provides the quantity ρ_E . Sometimes \underline{J} is taken out of \underline{j} (Clemmow and Dougherty, 1969). As expected

$$\begin{aligned} \text{div } \underline{j} &= \frac{\partial}{\partial t} (\underline{\nabla} \cdot \underline{P}) + \text{div } \underline{J} \\ &= - \frac{\partial \rho_p}{\partial t} - \frac{\partial \rho}{\partial t} \end{aligned} \quad (17)$$

where ρ_p = "equivalent" charge density due to polarisation in agreement with equation 11. To solve problems of phenomena purely internal to a plasma, the equations 12, 13, 14(a,b) and 15 are commonly used, putting $\underline{J} = 0$ and $\rho = 0$.

This paper treats a plasma in quasi-static conditions as a fluid with specifiable dielectric constant and magnetic permeability, using the unabridged Maxwell equations 1 to 7. The plasma is allowed to be driven from the outside by applied currents (\underline{J}) and charged distributions (ρ). This is in contrast to the present approach to plasma physics referred to above, which, in effect considers the charges and currents to be in a vacuum so that $\underline{D} = \underline{E}$ and $\underline{B} = \underline{H}$ (Clemmow and Dougherty, 1969), and the equivalent information on ϵ and μ is contained in velocity distribution functions and appropriate equations of motion.

Broad consistency between the two approaches is found; however this paper brings out new effects driven in a plasma by externally applied charge distributions which, in principle, could be quite arbitrary. Suspecting that many phenomena in the ionosphere, magnetosphere and interplanetary space are driven by outside sources we approach these phenomena theoretically in this way in the following discussion.

Electromagnetic Stress on a Dielectric Medium

All discussion from here on is restricted to quasi-static conditions. For slow variations of electric field orthogonal to \underline{B} , it is known that a plasma behaves like a dielectric medium. So that (Stix, 1962) for $\underline{B} = (0, 0, B_z)$

$$\underline{D} = (\epsilon E_x, \epsilon E_y, E_z) \quad (18)$$

where

$$\epsilon = 1 + \frac{4\pi\rho_m c^2}{B^2} \quad (19)$$

and

$$\rho_m = \sum_s n_s m_s. \quad (20)$$

n_s, m_s = number density, molecular mass respectively of species s of charged particle in the plasma.

Before discussing the macroscopic forces on a plasma let us refer to the classical discussion of the forces on a volume (V) of fluid with isotropic dielectric constant (Stratton, 1941). He shows that, given $\underline{B} = \mu \underline{H}$,

$$\frac{d}{dt} (\underline{G}_{\text{mech}} + \underline{G}_e) = \int_V \left[\rho \underline{E} + c^{-1} \underline{J} \times \underline{B} - \frac{1}{8\pi} E^2 \underline{\nabla} \epsilon - \frac{1}{8\pi} H^2 \underline{\nabla} \mu + \frac{1}{4\pi c} \frac{\partial}{\partial t} (\underline{D} \times \underline{B}) \right] dV, \quad (21)$$

where $\underline{G}_{\text{mech}}$ is the mechanical momentum of the volume of fluid and \underline{G}_e is the electromagnetic momentum in the volume (V). The components of the electric force $\rho \underline{E} - \frac{1}{8\pi} E^2 \underline{\nabla} \epsilon$ are given, in the case when $\underline{\nabla} \times \underline{E} = 0$, by

$$4\pi \rho E_j - \frac{1}{2} E^2 \frac{\partial \epsilon}{\partial x_j} = \sum_{k=1}^3 \frac{\partial}{\partial x_k} (E_j D_k) - \frac{1}{2} \frac{\partial}{\partial x_j} (\underline{E} \cdot \underline{D}). \quad (22)$$

In the steady state we may assume from equation 21 that

$$(4\pi)^{-1} (\text{div } \underline{D}) \underline{E} + c^{-1} \underline{J} \times \underline{B} - \frac{1}{8\pi} E^2 \underline{\nabla} \epsilon - \frac{1}{8\pi} H^2 \underline{\nabla} \mu = 0. \quad (23)$$

In addition one can say that the component of \underline{J} perpendicular to \underline{B} is

$$\underline{J} = \frac{c \underline{F} \times \underline{B}}{B^2}, \quad (24)$$

where

$$4\pi \underline{F} = (\text{div } \underline{D}) \underline{E} - \frac{1}{2} E^2 \underline{\nabla} \epsilon - \frac{1}{2} H^2 \underline{\nabla} \mu \quad (25)$$

Taking the stress tensor of equation 22 as applicable to a collisionless plasma (see Appendix) in which

$$\underline{B} = [0, 0, B_z(y)], \quad (26)$$

$$\underline{E} = [0, E_y(y), E_z], \quad (27)$$

$$\epsilon = \epsilon(y), \quad (28)$$

it is readily shown, as expected, that the electric force is given by

$$4\pi F_y^{el} = (\text{div } \underline{D}) E_y - \frac{1}{2} E_y^2 \frac{\partial E}{\partial y}, \quad (29)$$

$$F_z^{el} = (\text{div } \underline{D}) E_z / 4\pi. \quad (30)$$

Stratton (1941) points out that it is difficult to identify the mechanical and electric parts of the RHS of equation 21. However in the case of plasma this appears straightforward as follows. Firstly let us expand the right hand side as:

$$\int_V \left[\rho \underline{E} + \frac{1}{c} \underline{J} \times \underline{B} - \frac{1}{8\pi} E_{\perp}^2 \underline{\nabla} \epsilon - \frac{1}{8\pi} H^2 \underline{\nabla} \mu + \frac{1}{c} \frac{\partial \underline{P}}{\partial t} \times \underline{B} + \frac{1}{c} \underline{P} \times \frac{\partial \underline{B}}{\partial t} + \frac{1}{4\pi c} \frac{\partial}{\partial t} (\underline{E} \times \underline{B}) \right] dV. \quad (31)$$

Then $\rho \underline{E} - \frac{1}{8\pi} E_{\perp}^2 \underline{\nabla} \epsilon$ is the electric force; $\underline{J} \times \underline{B}$ the Lorentz force due to the current \underline{J} ; $-\frac{1}{8\pi} H^2 \underline{\nabla} \mu$ is $-\frac{1}{8\pi} \underline{\nabla} \left(\frac{P}{B^2} \right)$, part of the force due to $\underline{\nabla} p$, see equation 34; $\frac{\partial \underline{P}}{\partial t} \times \underline{B}$ is the Lorentz force due to the polarisation current, which is responsible for accelerating the plasma; $\frac{1}{c} \underline{P} \times \frac{\partial \underline{B}}{\partial t}$ being $-\underline{P} \times \underline{\nabla} \times \underline{E}$ is an additional force exerted by the field \underline{E} on the medium - this force has the dimensions, interestingly enough, of $(-\underline{\nabla} \cdot \underline{P}) \underline{E}$ but is different; notice that a force $(-\underline{\nabla} \cdot \underline{P}) \underline{E}$ does not appear in this approach, finally $\frac{1}{4\pi c} \frac{\partial}{\partial t} (\underline{E} \times \underline{B})$ is the rate of increase of the momentum of the electromagnetic field (c.f. Chandrasekhar, 1960). (see discussion later in this paper).

It may be noted that equation 21 was derived (Stratton, 1941) under the condition that μ is a function of position but not of field intensity. This is so in all applications in this paper. The value of μ in a 2-D collisionless plasma (see equation 68a) independent of B for the reason that p_{\perp}/B^2 is constant as B is changed in such a plasma (Chandrasekhar, 1960).

Let us now investigate the steady state flow of plasma as prescribed by equations 26, 27 and 28.

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Case I: Suppose $E_z \ll E_y$

The equations 9 and 24 yield

$$E_y \left[\frac{\partial}{\partial y} (\epsilon E_y) \right] - \frac{1}{2} E_y^2 \frac{\partial \epsilon}{\partial y} - \mu H_z \frac{\partial H_z}{\partial y} - \frac{1}{2} H_z^2 \nabla \mu = 0, \quad (32)$$

or

$$\frac{\partial}{\partial y} (\epsilon E_y^2) - \frac{\partial}{\partial y} (\mu H_z^2) = 0. \quad (33)$$

So

$$\epsilon E_y^2 - \frac{B_z^2}{\mu} = \text{constant}. \quad (34)$$

Equations 67 and 68 show that in the present special case

$$\mu = \frac{B_z}{H_z} = \frac{1}{1 + \frac{8\pi p_{\perp}}{B^2}}. \quad (35)$$

So equation 34 yields

$$\frac{\epsilon E_y^2}{8\pi} - \frac{B_z^2}{8\pi} - p_{\perp} = \text{constant} \quad (36)$$

Case II: No restriction on E_z except $\nabla \times \underline{E} = 0$.

Now equation 24 yields

$$E_y \left[\frac{\partial}{\partial y} (\epsilon E_y) + \frac{\partial}{\partial z} E_z \right] - \frac{1}{2} (E_y^2) \frac{\partial \epsilon}{\partial y} - \frac{1}{2} \frac{\partial}{\partial y} (H_z^2 \mu) = 0, \\ \frac{1}{2} \frac{\partial}{\partial y} (\epsilon E_y^2) - \frac{1}{2} \frac{\partial}{\partial y} (H_z^2 \mu) + E_y \frac{\partial E_z}{\partial z} = 0, \quad (37)$$

This predicts the natural existence of electrostatic fields E_z parallel to the magnetic field in plasmas.

It follows that

$$\frac{\partial E_z}{\partial z} = \frac{1}{E_y} \frac{\partial}{\partial y} (B^2 + 8\pi p - \epsilon E_y^2) \quad (38)$$

To calculate \underline{E} , ρ must first be reduced by $\underline{\nabla} \cdot \underline{P}$, to find $\text{div } \underline{E}$ (equation 11). To a high degree of approximation $\text{div } \underline{E} \approx 0$, otherwise intolerably large electric fields would exist. Therefore,

$$\frac{\partial E_z}{\partial z} = - \frac{\partial E_y}{\partial y} \quad (39)$$

Perhaps this is the simplest demonstration of the existence of significant electric fields parallel to \underline{B} in structures in which $\frac{\partial E_x}{\partial x} \approx 0$, and $\frac{\partial E_y}{\partial y}$ is non-zero and $\gg \frac{\partial E_x}{\partial x}$. The relationships 36 to 39 are testable, in principle, with measurements of plasma parameters in essentially laminar plasma flows in space above auroras, in the low latitude boundary layer of the magnetosphere and in flows in interplanetary space.

Still with conditions 26, 27 and 28 let us discuss the effect of F_z (equation 30). It is clear that because of the large values of ϵ that occur in plasmas (e.g., up to 10^6 in the magnetosphere and interplanetary space). The equation of motion of plasma along B_z may be written

$$\rho_m \frac{dv_z}{dt} = \left(\frac{\text{div } \underline{D}}{4\pi} \right) E_z - \rho_m g - \nabla p, \quad (40)$$

where g is the component of gravity along the field B_z . p is the pressure of the plasma measured parallel to \underline{B} . Clearly the effect of the term $(\text{div } \underline{D}) E_z / 4\pi$ can be significant in structures less than a certain cross section

(across \underline{B}). Usually when approximate neutrality is assumed in many plasma systems this term is neglected. However the term is of value $E_y \left[\frac{\partial}{\partial y} (\epsilon E_y) + \frac{\partial}{\partial z} E_z \right]$ in the case of flows with large variations of (ϵE_y) in the y direction, and correspondingly large contributions are made to F_z . Examples of this in geophysically significant situations are discussed later in this paper. This body force would cause jetting of the plasma along \underline{B} .

The Current Orthogonal to \underline{B}

Expressed generally by equations 25 and 26 the portion of \underline{J} driven by electric field, hereafter called the dielectric current, is given by

$$\underline{J}_{el} = \frac{c \left[(\text{div } \underline{D}) \underline{E} - \frac{1}{2} E^2 \underline{\nabla} \epsilon \right] \times \underline{B}}{4\pi B^2} . \quad (41)$$

In the special case given by equations 26, 27 and 28, this current is in the x -direction and of magnitude

$$J_{el} = \frac{c \frac{\partial}{\partial y} (\epsilon E_y^2)}{8\pi B^2} . \quad (42)$$

The total current density orthogonal to \underline{B} is, for the same geometry,

$$J_x = \frac{c \frac{\partial}{\partial y} [\epsilon E_y^2 - 8\pi p_{\perp}]}{8\pi B^2} . \quad (43)$$

The Current Parallel to \underline{B}

In addition to the jetting of plasma along \underline{B} mentioned earlier the parallel electric field will cause current parallel to \underline{B} . Such current would perturb the simple geometry prescribed by equations 26, 27 and 28. The analysis of complete current systems in general magnetic fields is a matter for later development.

A Numerical Illustration

Earlier the motions of single charged particles in a homogeneous magnetic field of induction \underline{B} and orthogonal electric field \underline{E} which has a gradient ∇E parallel to \underline{E} were studied (Cole, 1976). It was inferred that particles drift with velocity v_D , given here in c.g.s. units, by

$$v_{Ds} = \frac{cE}{B} \frac{\omega_s^2}{\Omega_s^2}, \quad (44)$$

producing a current in the direction $+\underline{E} \times \underline{B}$ depending on whether ∇E has the same or opposite direction to \underline{E} . (Note that the direction is more properly stated as $+\underline{E} \times \underline{B}$ rather than $+\nabla E \times \underline{B}$ as printed in that paper.)

Here $\omega_s = Be/m_s c$ and $\Omega_s^2 = \omega_s^2 - \frac{q_s \nabla E}{m_s c}$ when q_s is the charge on particles of mass m_s . In the case $\frac{q_s \nabla E}{m_s c} \ll \frac{B^2 q_s^2}{m_s^2}$, it is readily shown that

$$v_{Ds} \approx \frac{c^3 m_s}{2 q_s B^3} \nabla E^2 + \frac{cE}{B},$$

which is charge-dependent and mass-dependent.

In the special case of a plasma of uniform density this produces a current

$$\begin{aligned} j &= \sum_s n_s \frac{q_s}{c} v_{Ds} \\ &\approx \frac{\epsilon \nabla E^2}{8\pi B}, \end{aligned} \quad (45)$$

in agreement with the more general equation 42.

By comparison of the two approaches, in the case where ϵ is constant, equation 42 may be interpreted in terms of a drift for each charged particle additional to the well known drift $V_E = c(\underline{E} \times \underline{B}) B^{-2}$ of an amount

$$V_{\nabla E} = \frac{c^3 m_s \nabla E^2 \times \underline{B}}{2 q_s B^4}. \quad (46)$$

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It may be noted that V_{VE}/V_E is approximately the ratio of the gyroradius associated with a velocity V_E to the scale size of the spatial variation of ϵE_y .

In order to estimate the magnetic change due to the dielectric current, consider equation 42 in the form

$$\Delta B = E_y \Delta(\epsilon E_y). \quad (47)$$

So

$$\Delta B = \frac{E_y}{B} \Delta(\epsilon E_y).$$

The quantity $\epsilon E_y/B$ has the dimensions of velocity V_E . So we can write

$$\Delta B \approx \frac{V_E^2}{V_A^2} B, \quad (48)$$

where V_A is the Alfvén speed $= c/\epsilon^{\frac{1}{2}}$.

Alternatively

$$\Delta B \approx V_E^2 a \pi n_s m_s B^{-1}. \quad (49)$$

where $a = 2$ if $\Delta B/B \ll 1$, and $a = 4$ if $\Delta B \approx B$.

Table I shows estimates of upper limits to ΔB in various regions of interest.

TABLE I

Region	V_E (cms ⁻¹)	B (gauss)	Mi (amu)	Ni (cm ⁻³)	ΔB (gamma)
Low lat. Ionosphere*	10 ⁵ **	0.3	20	5x10 ⁶	4
Auroral Ionosphere*	4x10 ⁵	0.5	30	5x10 ⁶	50
Mid-Magnetosphere	3x10 ⁶	0.05	1.5	10 ³	0.3
Low lat. Boundary	1.5x10 ⁷	4x10 ⁻⁴	1.5	10	17
Layer	4x10 ⁷	4x10 ⁻⁴	1.5	10	~B
Interplanetary Space	4x10 ⁷	B	1.5	10	~B
Inner Solar Corona	10 ⁷	B	1.5	10	~B
Base of Corona		1	1.5	10 ⁸	~B

* Above altitude at which collision frequency $\nu = \omega_s$.
** Including corotation velocity with earth.

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The effects of dielectric currents are greatest in the boundary layer and in interplanetary space and the geophysical consequences of these currents will be dealt with in a separate paper. The currents may be detectable in the ionosphere particularly in polar regions when the highest spatial variations of the quantity ϵE_y occur.

It is clear from Table I that the currents must be taken into account in discussions of the physics of the low latitude boundary layer, interplanetary medium and the solar corona where the effects of dielectric current can be large.

The current cm^{-1} integrated in the y direction through the model is, from equation 42,

$$j(\text{cm}^{-1}) = \Delta(\epsilon E_y^2)/8\pi B \text{ emu}, \quad (50)$$

$$= n_i m_i V_E^2/2B. \quad (51)$$

In the case of gradient of electric fields above, or in the vicinity of auroras, we assume $n_i = 5 \times 10^6 \text{ cm}^{-3}$, $m_i = 30 \text{ amu}$ (i.e. NO^+), $B = 0.5 \text{ g}$, $V_E = 4 \times 10^5 \text{ cm/sec}$. These are extreme values. From equation 45 then $j \approx 1.6 \times 10^{-4} \text{ amp cm}^{-1}$. Integrating over the height of the ionosphere where the ions are freely gyrating, i.e., say 180-400 km altitude the total current would be $3.5 \times 10^3 \text{ amp}$. In the first approximation the altitude variation of the current density is like that of the ion mass density. A significant effect should hold from 150 km altitude to the altitude of transition of heavy ions (NO^+ , O^+) to light ones (He^+ , H^+). Sometimes above auroras this transition can be very high in which case the total current could be greater.

In the low latitude boundary layer (Cole, 1974; Eastman and Hones, 1979) in which, e.g., $B = 5 \times 10^{-4}$, $V_E = 2 \times 10^{-7} \text{ cm}^{-1}$, $n_i = 10 \text{ cm}^{-3}$ of H^+ plus

3 cm^{-3} of He^+ the current integrated through the layer is $\sim 1.83 \times 10^{-5}$ emu = 1.83×10^{-4} amp cm^{-1} . If the boundary layer extends five earth radii above and below the equatorial plane, then the total dielectric current flowing in the boundary layer would be $\sim 1.2 \times 10^6$ amps. For uncomplicated anti-solar flow and velocity gradient pointing outwards, the current is towards the sun on the dusk side and away from the sun on the dawn side. On both sides the Lorentz force of the current is inwards to the magnetosphere. The currents can be of magnitude comparable to the conventional Chapman-Ferraro currents. Fuller discussions of these problems and applications in the solar wind and other astrophysical situations will be taken up in later papers.

Effects Parallel to \underline{B}

With the model prescribed by equations 26, 27 and 28, the body force on the plasma parallel to \underline{B} is

$$F_z = E_z \frac{\partial}{\partial y} (\epsilon E_y) / 4\pi \quad (52)$$

The divergence of the displacement vector \underline{D} due to spatial variations in the y direction together with the large values of ϵ in plasmas create large net charge density, acting upon which, E_z can give a large rate of change of momentum to the plasma in the z direction.

In general we can say that the net acceleration of the plasma is given by

$$\rho_m \frac{dv_{\parallel}}{dt} = E_{\parallel} [(4\pi)^{-1} \frac{\partial}{\partial y} (\epsilon E_y)] - \rho_m g_{\parallel} - \nabla p_{\parallel} \quad (53)$$

It is of interest to find what length scale (L^*) of variation of (ϵE_y) in the y direction would yield the same acceleration per ionic mass in equation 53 as $e E / m_i$. It is given by

$$L^* = \frac{c^2 E_y}{e B^2} = \frac{m_i V_E c}{B e} . \quad (54)$$

L^* is the gyroradius of an ion moving \perp to \underline{B} with a velocity of V_E . Table II shows L^* for various regions of interest geophysically.

TABLE II

Regions	V_E (cms $^{-1}$)	B(gauss)	M_i (amu)	L^* (cm)
Low lat. Ionosphere	10^5	0.3	20	6.6×10^2
Polar Ionosphere	4×10^5	0.5	20	2.6×10^3
Mid-Magnetosphere	3×10^6	0.05	1.5	3×10^3
Outer Magnetosphere	1×10^7	0.005	1.5	3×10^5
Low lat. Boundary Layer	1.5×10^7	0.0004	1.5	5.6×10^6
Interplanetary Space	4×10^7	0.00005	1.5	1.2×10^8

Structures with scale sizes less than these, in the presence of electric fields orthogonal to \underline{B} would necessarily exhibit plasma flows along \underline{B} , according to equation 53, at places of strong gradient of (ϵE_y) . This would occur naturally at the sharp edges of auroral forms, especially fine auroral rays. Such flows may also be induced in the presence of appropriate electric fields by chemical releases, e.g. Ba, Li, and discharges from charged-particle guns. From 53 it may be inferred that, in a steady state, the distribution of plasma along \underline{B} would achieve a distribution given by

$$\nabla_{\parallel} p = -\rho_m g_{\parallel} + E_{\parallel} [(4\pi)^{-1} \frac{\partial}{\partial y} (\epsilon E_y)] . \quad (55)$$

The Momentum Per Unit Mass Orthogonal to \underline{B} (when $p_{\perp} = 0$)

It is clear from the examination of particle motions (Cole, 1976) that the current (equation 39) is due to ions in proportion to their contribution

to the mass density of the plasma. When $\frac{\partial}{\partial y} (\epsilon E_y^2) = 0$, the drift of the plasma orthogonal to \underline{B} , caused by an electric field, is given in a collisionless plasma, by

$$\underline{V} = \frac{c \underline{E} \times \underline{B}}{B^2}. \quad (56)$$

However in the circumstances given by equations 26, 27 and 28, neglecting the momentum of electrons, the momentum per unit mass is given by

$$V_x = \frac{c}{B} \left(E_y + \frac{1}{en} \frac{\partial (\epsilon E_y^2)}{8\pi \partial y} \right). \quad (57)$$

This result means that only if $\frac{\partial}{\partial y} (\epsilon E_y^2) \ll 8\pi en E_y$ may it be stated that, approximately,

$$c \underline{E} + \underline{V} \times \underline{B} = 0 \quad (58)$$

In this approximation equation 34 becomes, when $\epsilon \gg 1$.

$$p_{\perp} + \frac{B_z^2}{8\pi} - \frac{1}{2} \rho_m V_x^2 = \text{constant}, \quad (59)$$

which is a Bernoulli-like relationship, but pertaining to a line orthogonal to the flow.

The Force - $\underline{P} \times \underline{\nabla} \times \underline{E}$

This force which was uncovered in equation 31 makes sense as the electrical counterpart of the force - $\underline{B} \times \underline{\nabla} \times \underline{H}$. Both may be described as forces originating from dipoles. The fact that there is no magnetic counterpart of $(4\pi)^{-1} (\text{div } \underline{D}) \underline{E}$ is related to the lack of magnetic monopoles in the theory.

Self-Consistency

The self-consistency of the method may be verified by calculating the forces (electric and magnetic) on the plasma in any uniformly moving frame of reference including one moving with the plasma velocity (equation 57).

Consider a frame moving with uniform velocity \underline{V} with respect to the laboratory (L). Then (Jones, 1964)

$$\underline{D} = \underline{D}_L + \underline{V} \times \underline{H}/c, \quad (60)$$

$$\underline{E} = \underline{E}_L + \underline{V} \times \underline{B}/c. \quad (61)$$

Suppose $V \ll c$, so $\underline{B} \approx \underline{B}_L$. Suppose the plasma is cold i.e., $\mu = 1$, so that $\underline{B} = \underline{H}$. Also suppose $\epsilon = \text{constant}$. For laminar flow such as implied by equation 57

$$\text{div } \underline{D} = \text{div } \underline{D}_L \quad (62)$$

i.e., charge density is the same in both frames. The electric force in a frame moving with velocity \underline{V} multiplied by 4π is

$$(\text{div } \underline{D}) \underline{E} = (\text{div } \underline{D}_L) \underline{E}_L + (\text{div } \underline{D}_L) \underline{V} \times \underline{B}/c. \quad (63)$$

The current in the frame moving with velocity \underline{V} is

$$\underline{J} = \underline{J}_L - \left(\frac{\text{div } \underline{D}}{4\pi} \right) \underline{V}. \quad (64)$$

So the magnetic force in the moving frame is

$$\underline{J} \times \underline{B}/c = \underline{J}_L \times \underline{B}/c - \left(\frac{\text{div } \underline{D}}{4\pi} \right) \underline{V} \times \underline{B}/c \quad (65)$$

But

$$\left(\frac{\text{div } \underline{D}_L}{4\pi} \right) \underline{E}_L = - \underline{J}_L \times \underline{B}/c \quad (66)$$

So

$$\left(\frac{\text{div } \underline{D}}{4\pi}\right) \underline{E} = - \underline{j} \times \underline{B}/c.$$

Consequently there is a balance of forces in all inertial frames. The method can claim consistency with the Lorentz invariance of Maxwell's equations.

The electric currents and forces in a plasma revealed by this method come about by allowing the plasma to be driven by external sources and treating these sources according to Maxwell's unabridged equations.

The Magnetic Permeability of a 2-D Collisionless Plasma

Consider an infinite two dimensional collisionless plasma in which $\underline{B} = (0, 0, B_z(y))$. In a steady state in the absence of electric field, equation 31 yields,

$$(\underline{\nabla} \times \underline{H}) \times \underline{B} - \frac{1}{2} H^2 \underline{\nabla} \mu = 0, \quad (67)$$

which in the assumed geometry becomes

$$-\mu H \frac{\partial H}{\partial y} - \frac{1}{2} H^2 \nabla \mu = 0,$$

or

$$\frac{\partial}{\partial y} (\mu H^2) = 0.$$

Therefore

$$\mu H^2 = K = B^2/\mu.$$

where K is a constant.

Also $H = K/B$.

In the case of collisionless plasma let us assume that

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$$K = B^2 + 8\pi p_{\perp}.$$

This assumption can be proved in the case of low β plasma as is done below, and of course K is a well known constant of 2-D collisionless plasma. Then,

$$\mu = \frac{1}{1 + \frac{8\pi p_{\perp}}{B^2}}, \quad (68)$$

because $\mu = 1$, when $p_{\perp} = 0$.

This defines the vectors \underline{H} and \underline{M} by

$$\underline{H} = \left(1 + \frac{8\pi p_{\perp}}{B^2}\right) \underline{B} = \frac{K}{B^2} \underline{B}$$

and

$$\underline{M} = \left(-\frac{2p_{\perp}}{B^2}\right) \underline{B} = \frac{1}{4\pi} \left(1 - \frac{K}{B^2}\right) \underline{B}$$

The magnetisation vector \underline{M} is twice the "conventional" magnetisation vector and presumably incorporates equal components of the diamagnetism due to gyration and due to grad \underline{B} drifting. The total current in the plasma is the sum of $(4\pi)^{-1} c \nabla \times \underline{H}$ and $c(\nabla \times \underline{M})$, i.e., $(4\pi)^{-1} c \nabla \times \underline{B}$. In the present geometry this total plasma current is then

$$\frac{c}{4\pi} \frac{\partial B}{\partial y} = -\frac{c}{B} \frac{\partial p_{\perp}}{\partial y},$$

which is the same as that calculated from particle motions in the field \underline{B} (Chandrasekhar, 1960).

Consistency with equation (a) is observed because, in this case

$$(\underline{\nabla} \times \underline{H}) \times \underline{B} = \frac{K}{4\pi B} \frac{\partial B}{\partial y} = - \frac{1}{8\pi} B^2 \frac{\partial}{\partial y} \left(\frac{1}{\mu} \right),$$

where

$$\frac{1}{\mu} = K/B^2.$$

In the case of low β plasma, $\underline{H} \approx \underline{B}$ and for present geometry,

$$\underline{\nabla} \times \underline{H} = \underline{\nabla} \times \underline{B} = - \frac{4\pi}{B} \frac{\partial p_{\perp}}{\partial y}$$

Therefore, equation (a) yields

$$\begin{aligned} - \frac{\partial p_{\perp}}{\partial y} &= \frac{1}{8\pi} H^2 \nabla_{\mu}, \\ &= - \frac{1}{8\pi} B^2 \frac{\partial}{\partial y} \left(\frac{1}{\mu} \right). \end{aligned}$$

Therefore,

$$\frac{\partial \left(\frac{1}{\mu} \right)}{\partial p_{\perp}} = \frac{8\pi}{B^2},$$

which leads to the result

$$\frac{1}{\mu} = 1 + \frac{8\pi p_{\perp}}{B^2}. \quad (63a)$$

It may be of interest to note that this value can be used with the classical relationship for the phase velocity of an electromagnetic wave in a non-dissipative medium, viz.,

$$V_{ph}^2 = c^2/\epsilon\mu. \quad (69)$$

Microscopic plasma physics shows, for a magnetosonic wave, (Stix, 1962) moving orthogonal to \underline{B} , exactly the same result, i.e.,

$$c^2 v_{ph}^{-2} = \frac{1 + 4\pi\rho_m c^2/B^2}{1 + \beta_{\perp}}, \quad (70)$$

where $\beta_{\perp} = \frac{8\pi p_{\perp}}{B^2}$.

Discussion

The dielectric current orthogonal to a magnetic field in a plasma has been put on a general basis. An earlier discussion of the movement of charged particles in crossed magnetic and spatially varying electric fields revealed a special case of it (Cole, 1976). The current comes about principally on account of the large value of the dielectric constant of a plasma. Structures exist in plasmas in the earth's environment in which the current plays a significant role, e.g., in the auroral ionosphere and the low latitude boundary layer. In a later paper the significance of the current in interplanetary space will be demonstrated. This new current should be sought experimentally and taken into account theoretically in models of plasma structures.

The dielectric property of plasma also leads to acceleration of plasma bodily along the magnetic field at places of fine structure of the quantity $\epsilon \underline{E}$ and to a new distribution of plasma along \underline{B} .

The coexistence of electric field parallel and perpendicular to \underline{B} in plasma flows of special geometry has been demonstrated. Some flows in nature appear to have similar geometry. In later papers the consequences of the field and currents in the low latitude boundary layer and in interplanetary space and the solar corona will be discussed.

These new results are not evident in the widespread approach to plasma physics which focusses on internal processes and their description in terms of \underline{E} and \underline{B} , rendering, for that purpose, \underline{D} and \underline{H} apparently unnecessary as vehicles in analyses. However by such examination of internal processes, values of ϵ and μ can be established. Having these values at our disposal allows us to regard a plasma as a medium, with specifiable ϵ and μ , which can be acted upon from the outside by electromagnetic fields. To analyse such action requires the unabridged Maxwell equations employing \underline{D} , \underline{E} , \underline{B} and \underline{H} .

Not everything which happens in a plasma is the result of an internal instability. E.g., the magnetosphere is an intermediary medium between the solar wind "driver" and the ionosphere, sometimes also a "driver". Energy is transported across the magnetosphere on time scales far in excess of the time it takes for Alfvén waves to traverse the system. It is clear that to study quasi-steady state macroscopic phenomena such as large scale current systems and plasma flows in the magnetosphere, the approach as outlined in this paper will need to be taken. The same argument can be made of numerous other systems in interplanetary space and stellar atmospheres.

In the (\underline{D} , \underline{E} , \underline{B} , \underline{H}) approach to plasmas, all the internal currents produced by properties of the medium were lumped together in determining μ . Thus, in the model defined by equations 26, 27 and 28, a value of μ was used which combined into the vector $\underline{\nabla} \times \underline{M}$ both the conventional "magnetisation" current of plasma physics and the gradient \underline{B} current in a plasma (Chandrasekhar, 1960) i.e., all the currents related to ion and electron pressure in the specified geometry. The value of ϵ is taken from standard plasma physics. Parameterising a plasma in this way may facilitate, in some circumstances, the macroscopic description of phenomena in it driven from the outside.

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The virtue of the approach to plasmas in this paper is that it allows an internally consistent discussion of non-linear effects in plasmas. However, it has relied upon linearised equations (e.g., Stix, 1962) to provide values of ϵ and μ . There is a sense in which one can speak of a collisionless plasma as a perfect Maxwell medium in which the charged particles and the fields, \underline{D} , \underline{E} , \underline{B} , \underline{H} , mutually interacting determine the behavior of the medium. There are no internal strains to be considered as there are in the cases, e.g., of solid dielectrics and ferromagnets (see Stratton, 1941). It is this sense which has allowed the interpretation of equation 21 which has been developed in this paper.

It is now possible to see clearly the relationship of the approach of this paper to the conventional approach defined by equations 8, 12, 13, 14 (a and b) and 15. By assuming $\underline{J} = 0$ the conventional approach immediately allows only the existence of polarisation current and magnetisation current (see equation 12). The new current defined by equations 24 and 25 is neither of these, but is derived from more general considerations. It should be appreciated then, that, that approach, though powerful, has its limitations. The approach of this paper extends our ability to describe plasma phenomena.

Generalisation

Having established the utility of the (\underline{D} , \underline{E} , \underline{B} , \underline{H}) approach, and predicted new results with it in special geometry, and observed its consistency with other approaches to plasma physics, it is clear that new general results can be established by returning to equation 21 and rewriting it as

$$\frac{d}{dt} (\underline{G}_{\text{mech}} + \underline{G}_e) = \int_V [\nabla \cdot (\psi_{e1} + \psi_{\text{mag}})] dV, \quad (71)$$

where ψ_{el} is the electric stress tensor and ψ_{mag} the magnetic one, given by

$$4\pi \psi_{elij} = E_i D_j - \frac{1}{2} (\underline{E} \cdot \underline{D}) \delta_{ij}, \quad (72a)$$

$$4\pi \psi_{magij} = H_i B_j - \frac{1}{2} (\underline{H} \cdot \underline{B}) \delta_{ij}. \quad (72b)$$

δ_{ij} is the Kronecker delta.

In the most general case assume, using direction cosine notation, that

$$\underline{E} = E(e_x, e_y, e_z), \quad (73)$$

and

$$\underline{B} = B(b_x, b_y, b_z). \quad (74)$$

where

$$e_x^2 + e_y^2 + e_z^2 = b_x^2 + b_y^2 + b_z^2 = 1.$$

Since

$$\underline{D} = \epsilon \underline{E}_\perp + E_\parallel \quad (75)$$

where \parallel, \perp mean components parallel and perpendicular, respectively, to \underline{B}

$$\underline{D} = E[\epsilon e_x(1-b_x) + b_x e_x, \epsilon e_y(1-b_y) + b_y e_y, \epsilon e_z(1-b_z) + b_z e_z] \quad (76)$$

Then,

$$\psi_{elxx} = \frac{1}{8\pi} E^2 \left\{ e_x^2 [\epsilon(1-b_x) + b_x] - e_y^2 [\epsilon(1-b_y) + b_y] - e_z^2 [\epsilon(1-b_z) + b_z] \right\} \quad (77)$$

$$\psi_{elxy} = \frac{1}{4\pi} E^2 e_x e_y [\epsilon(1-b_y) + b_y] \quad (78)$$

$$\psi_{elxz} = \frac{1}{4\pi} E^2 e_x e_z [\epsilon(1-b_z) + b_z] \quad (79)$$

$$\psi_{elyx} = \frac{1}{4\pi} E^2 e_y e_x [\epsilon(1-b_x) + b_x] \quad (80)$$

$$\begin{aligned} \psi_{elyy} = \frac{1}{8\pi} E^2 \left\{ e_y^2 [\epsilon(1-b_y) + b_y] - e_z^2 [(1-b_z) + b_z] \right. \\ \left. - e_x^2 [\epsilon(1-b_x) + b_x] \right\} \end{aligned} \quad (81)$$

$$\psi_{elyz} = \frac{1}{4\pi} E^2 e_y e_z [\epsilon(1-b_z) + b_z] \quad (82)$$

$$\psi_{elzx} = \frac{1}{4\pi} E^2 e_z e_x [\epsilon(1-b_x) + b_x] \quad (83)$$

$$\psi_{elzy} = \frac{1}{4\pi} E^2 e_z e_y [\epsilon(1-b_y) + b_y] \quad (84)$$

$$\begin{aligned} \psi_{elzz} = \frac{1}{8\pi} E^2 \left\{ e_z^2 [\epsilon(1-b_z) + b_z] - e_x^2 [\epsilon(1-b_x) + b_x] \right. \\ \left. - e_y^2 [\epsilon(1-b_y) + b_y] \right\} \end{aligned} \quad (85)$$

while

$$\psi_{magxx} = \frac{1}{8\pi\rho} (B_x^2 - B_y^2 - B_z^2) \quad (86)$$

$$\psi_{magxy} = \frac{1}{4\pi\mu} B_x B_y \quad (87)$$

$$\psi_{magxz} = \frac{1}{4\pi\mu} B_x B_z \quad (88)$$

$$\psi_{magyx} = \frac{1}{4\pi\mu} B_y B_x \quad (89)$$

$$\psi_{magyy} = \frac{1}{8\pi\mu} (B_y^2 - B_z^2 - B_x^2) \quad (90)$$

$$\psi_{magyz} = \frac{1}{4\pi\mu} B_y B_z \quad (91)$$

$$\psi_{magzx} = \frac{1}{4\pi\mu} B_z B_x \quad (92)$$

$$\psi_{magzy} = \frac{1}{4\pi\mu} B_z B_y \quad (93)$$

$$\psi_{\text{magzz}} = \frac{1}{8\pi\mu} (B_z^2 - B_x^2 - B_y^2) \quad (94)$$

The three relations that are obeyed by a plasma in a steady state are then

$$\underline{\nabla} \cdot (\psi_{\text{el}} + \psi_{\text{mag}}) = 0. \quad (95)$$

Equation 34 is found when $\underline{\nabla} \times \underline{E} = b_x = b_y = c_x = c_z = 0$. Satellite borne equipment now measures all three components of \underline{E} and \underline{B} as well as plasma properties from which ϵ and μ may be evaluated. So equation 95, using equations 77-94 may be tested in steady state structures. The full testing of the consistency of measurements of fields \underline{E} and \underline{B} and plasma parameters ϵ and μ against theory for quasi-static structures would need to allow for the possibility of slow but measureable variations in them. However there would be a difficult experimental problem of distinguishing between temporal and spatial variations. For this reason it is necessary to have measurements of fields and plasma properties at many different places simultaneously to properly understand the physics of auroral and space plasmas.

It is of interest to expand equation 71, thus

$$\begin{aligned} \frac{d}{dt} (G_{\text{mech}} + G_c) = \int_V \left[\left(\frac{\text{div } \underline{D}}{4\pi} \right) \underline{E} + c^{-1} \underline{J} \times \underline{B} - \frac{1}{8\pi} \underline{E}_\perp^2 \underline{\nabla} \epsilon \right. \\ \left. - \frac{1}{8\pi} H^2 \underline{\nabla} \mu + \frac{\partial}{\partial t} (\underline{D} \times \underline{B}) \right] dV, \end{aligned} \quad (96)$$

and

$$\begin{aligned} \frac{\partial}{\partial t} (\underline{D} \times \underline{B}) = \frac{\partial}{\partial t} (\underline{E} \times \underline{H}) - 4\pi \underline{M} \times \frac{\partial \underline{E}}{\partial t} + 4\pi \underline{E} \times \frac{\partial \underline{M}}{\partial t} \\ + 4\pi \frac{\partial \underline{P}}{\partial t} \times \underline{B} - 4\pi \underline{P} \times \underline{\nabla} \times \underline{E} \end{aligned} \quad (97)$$

$$= \frac{\partial}{\partial t} (\underline{E} \times \underline{B}) + 4\pi \frac{\partial \underline{P}}{\partial t} \times \underline{B} - 4\pi \underline{P} \times \underline{\nabla} \times \underline{E} \quad (98)$$

The last two terms of equations 97 and 98 have been commented on earlier in this paper. Chandrasekhar (1962) calls $\frac{1}{4\pi c} \frac{\partial}{\partial t} (\underline{E} \times \underline{B})$ the rate of change of electromagnetic momentum. This term is a combination of $\frac{1}{4\pi c} \frac{\partial}{\partial t} (\underline{E} \times \underline{H})$, which is the Abraham and von Laue expression for rate of change of electromagnetic momentum in material media (Stratton, 1941), plus forces on the medium due to the interaction of the electric field (\underline{E}) and the magnetisation field (\underline{M}) of amount $\frac{1}{c} \frac{\partial}{\partial t} (\underline{E} \times \underline{M})$. This corresponds to the interpretation of $\frac{1}{4\pi c} \underline{E} \times \underline{H}$, as the Poynting vector. In slow time variations as envisaged in this paper, this author considers that $\frac{1}{c} \frac{\partial}{\partial t} (\underline{E} \times \underline{M})$ should be interpreted as a force on the medium contributing to its acceleration.

For variations in the plasma, slow compared to the ion gyroperiod, the forces $\frac{1}{c} \frac{\partial}{\partial t} (\underline{E} \times \underline{M})$ and $\frac{1}{c} \frac{\partial}{\partial t} (\underline{P} \times \underline{B})$ may be considered to be equivalent in their mechanical effects to a current in the plasma of amount given by

$$\underline{j}^* = \left[\frac{\partial}{\partial t} (\underline{E} \times \underline{M} + \underline{P} \times \underline{B}) \right] \times \underline{B} B^{-2}. \quad (99)$$

This expression contains, as one component, the well-known polarisation current. The three new currents implied by equation 99 are

$$\underline{j}_1^* = B^{-2} \underline{B} \times \left(\underline{M} \times \frac{\partial \underline{E}}{\partial t} \right), \quad (100)$$

$$\underline{j}_2^* = B^{-2} \left(\underline{E} \times \frac{\partial \underline{M}}{\partial t} \right) \times \underline{B}, \quad (101)$$

$$\underline{j}_3^* = c B^{-2} \underline{B} \times (\underline{P} \times (\underline{\nabla} \times \underline{E})). \quad (102)$$

The whole of \underline{j}^* would be in $\frac{\partial \underline{D}}{\partial t}$ and \underline{J} of equation 2. The absence of the current of equations 100, 101 and 102 in much conventional plasma physics which arbitrarily puts $\underline{J} = 0$ should give us pause to consider the assumptions of that method. Of course \underline{j}_1^* and \underline{j}_2^* are zero in a cold plasma while \underline{j}_3^*

is $\frac{cP}{E} \cdot \frac{\Delta B}{\Delta P}$ times the well-known polarisation current where Δ denotes perturbation (by a wave, say).

Now

$$\frac{cP}{B} \cdot \frac{\Delta B}{\Delta P} \approx \frac{cE}{B} \cdot \frac{\Delta B}{\Delta E}$$

$$\approx V_E/V_{ph}$$

where V_{ph} is the phase velocity of the wave. So, in many circumstances j_3^* will be a negligible current, but not always. Likewise, if we compare j_1^* to $c \nabla \times \underline{M}$ the ratio is $V_{ph} V_E \Delta E/(EC^2)$, again often negligible, but not always. As regards the ratio of j_2^* to $c \nabla \times \underline{M}$, it is $V_{ph} V_E \Delta M/(MC^2)$ which is often negligible.

Accepting the Abraham and von Laue expression for electromagnetic momentum, even in a material medium (Stratton, 1941), the rate of change of mechanical momentum per unit volume of plasma would be given by the equation,

$$\begin{aligned} \frac{d}{dt} (\rho_m \underline{V}) = & \rho \underline{E} + \frac{1}{c} \underline{J} \times \underline{B} - \frac{1}{8\pi} E^2 \nabla \epsilon - \frac{1}{8\pi} H^2 \nabla \mu \\ & + \frac{1}{c} \frac{\partial P}{\partial t} \times \underline{B} + \frac{1}{c} \underline{P} \times \frac{\partial \underline{B}}{\partial t} + \frac{1}{c} \underline{E} \times \frac{\partial \underline{M}}{\partial t} - \frac{1}{c} \underline{M} \times \frac{\partial \underline{E}}{\partial t} \end{aligned} \quad (103)$$

The conclusions expressed in equations 99-102, based upon the Abraham and von Laue expression for electromagnetic momentum would be important in various circumstances, e.g., the current j_3^* could be significant in collisionless shocks, and the currents j_1^* and j_2^* in relativistic plasmas.

It should be noted that equation 95 is a "weak" condition for a steady state because it does allow the exchange of momentum between mechanical and electromagnetic components within the volume considered. From equation 103 and equation 2 it can be inferred that a "mechanical" steady state of the

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plasma is acquired when the following stronger conditions exists

$$(\text{div } \underline{D}) \underline{E} + (\underline{\nabla} \times \underline{H}) \times \underline{B} - \frac{1}{2} E_{\perp}^2 \underline{\nabla} \epsilon - \frac{1}{2} H^2 \underline{\nabla} \mu = 0 \quad (104)$$

For the conditions prescribed by equations 73, 74 and 75, equation 104 yields

$$\begin{aligned} & \left[\frac{\partial}{\partial x} (\epsilon E_x) + \frac{\partial}{\partial y} (\epsilon E_y) + \frac{\partial}{\partial z} E_z \right] E_x - \frac{1}{2} (E_x^2 + E_y^2) \frac{\partial \epsilon}{\partial x} \\ & + \frac{1}{2} B^2 \frac{\partial}{\partial x} \left(\frac{1}{\mu} \right) - B_z \left(\frac{\partial (B_z/\mu)}{\partial x} - \frac{\partial (B_x/\mu)}{\partial z} \right) \\ & + B_y \left(\frac{\partial (B_x/\mu)}{\partial y} - \frac{\partial (B_y/\mu)}{\partial x} \right) \end{aligned} \quad (105)$$

and similar equations for the y and z components of 104. It is clear that these can be integrated only for the simplest of geometry such as that of equations 26, 27 and 28. From an experimentalist's viewpoint this would mean that only with the simplest of geometry may the basic equations be tested in the steady state, in integral form, using single point measurements. The alternative is to test them in differential form (like equations 104). In space physics terms this calls for clusters of satellites or space probes identically instrumented to study space plasmas definitively.

There is much discussion in the literature as to the appropriateness of $\frac{\underline{E} \times \underline{H}}{4\pi c}$ (the Abraham and von Laue expression) or $\frac{\underline{D} \times \underline{B}}{4\pi c}$ (the Minkowski expression) for the electromagnetic momentum in a material medium (see Møller, 1972). Møller suggests that it may not be possible to resolve this question. The question may be more than one of the semantics of equivalent descriptions of forces on a system consisting of electromagnetic fields and matter. In the case of plasma in a magnetic field the so-called "polarisation current" is clearly seen to play a role in accelerating plasma (c.f., Spitzer, 1962). But the force due to the "polarisation current" is $\frac{1}{c} \frac{\partial \underline{P}}{\partial t} \times \underline{B}$ and this is part

of $\frac{1}{4\pi c} \frac{\partial}{\partial t} (\underline{D} \times \underline{B})$. This indicates that when we come to discuss the mechanical momentum of a plasma the semantic dilemma is broken and that the Abraham and von Laue expression for electromagnetic momentum is valid. This points to the reality of the new currents implied by equation 99. Moreover this discussion points to the value of space plasma measurements in answering fundamental questions in electromagnetism, because in collisionless plasmas, not only can the fields and particle distributions now be measured, but this medium has the virtue of the lack of additional internal forces to contend with. What is lacking at present is measurements of such plasma with sufficient spatial and temporal resolution.

Finally it should be mentioned that it is expected that the new current (equation 42) should also exist in collision dominated plasma because a gradient of electric field energy density is still equivalent to a force on the plasma. Further, it should be understood that the currents uncovered in this paper must imply that stability theory of plasma is far from complete and these currents should be taken into account in such theory.

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Appendix

The stress tensor

As Stratton (1941) explains, the stress tensors would be different from those of equations 72a and 72b, if ϵ were a function of E , and μ a function of B . Since

$$\epsilon = 1 + \sum_n \frac{4\pi n_s m_s c^2}{B^2},$$

ϵ is not a function of E and the electric field energy density is given by

$$u = \int_0^D \frac{\underline{E} \cdot d\underline{D}}{8\pi} = \epsilon \int_0^E \frac{E dE}{8\pi} = \frac{\epsilon E^2}{8\pi}.$$

Also since $\frac{1}{\mu} = 1 + \frac{8\pi p_{\perp}}{B^2} = 1 + \beta_{\perp}$, μ is not a function of B for 2-dimensional adiabatic changes, because, in such changes, n/B is constant and T_{\perp}/B is constant (Chandrasekhar 1960).

It follows that the magnetic energy density is given by

$$w = \int_0^B \underline{H} \cdot d\underline{B} = \frac{1}{\mu} \int_0^B \underline{B} \cdot d\underline{B} = \frac{B^2}{\mu}.$$

Therefore, it is appropriate to employ the stress tensors 72a and 72b in the case of 2-D collisionless plasmas.